

Production function with all variable input - Laws of Returns to Scale

The **law of returns to scale** explains the proportional change in output with respect to proportional change in inputs.

The laws of returns to scale explain how a simultaneous and proportionate increase in both labour (L) and capital (K) affects the total output at various levels of input combination.

The degree of change in output varies with change in the amount of inputs. For example, an output may change by a large proportion, same proportion, or small proportion with respect to change in input.

$$Q_x = f(K, L)$$

$$\begin{aligned} kQ_x &= f(kK, kL) \\ &= k f(K, L) \\ &= kQ_x \end{aligned}$$

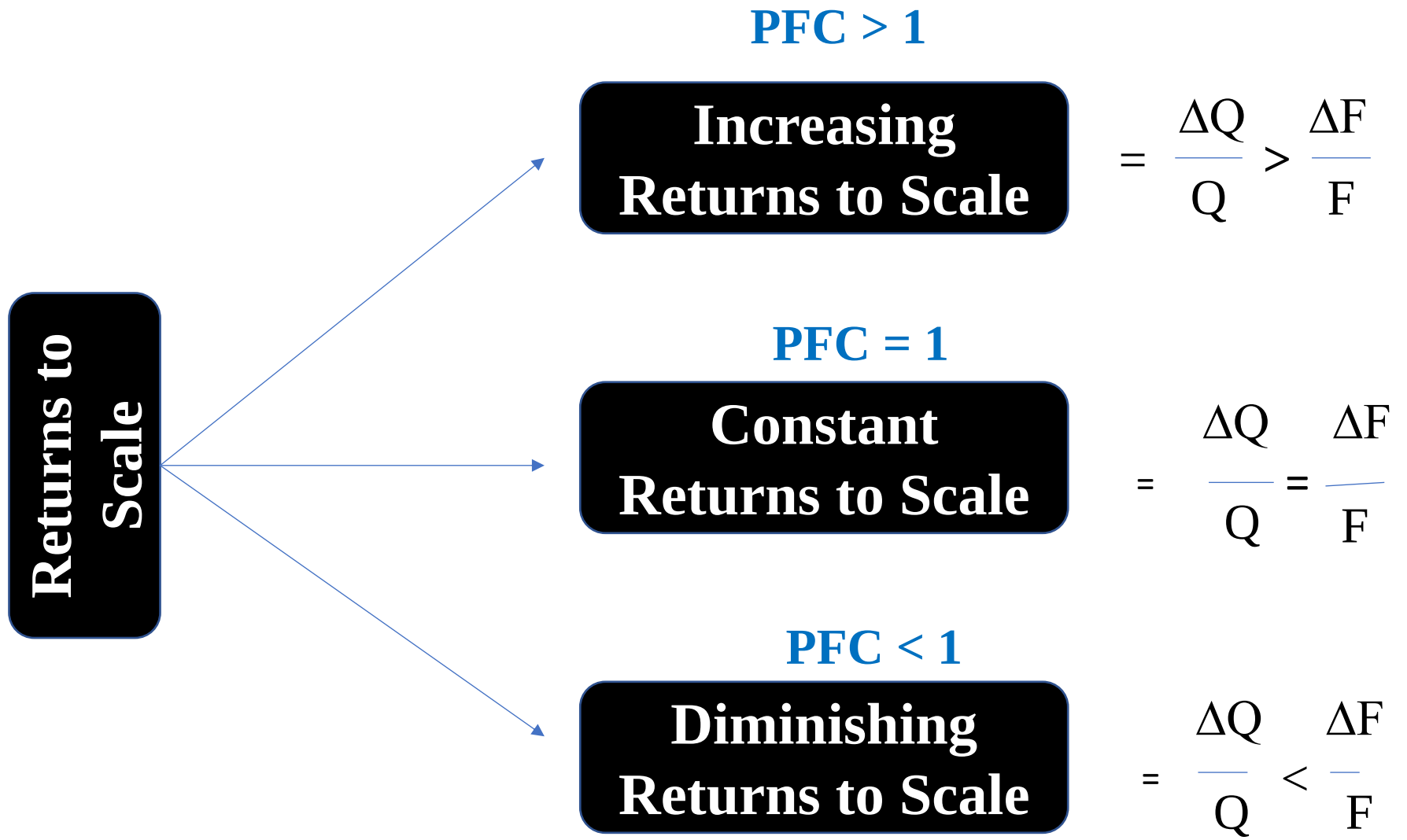
(Homogeneous Production Function)

$$hQ_x = f(kK, kL)$$

- The concept of **returns to scale** is a long-run phenomenon.
- The laws of returns to scale explain *the behaviour of the total output in response to changes in the scale of the firm. In response to a simultaneous and proportional increase in all the inputs* it used.
- More precisely, *the laws of returns to scale explain how a simultaneous and proportionate increase in both labour (L) and capital (K) affects the total output at various levels of input combination.*

- *When a firm increases all its inputs proportionately, technically*, there are three possibilities i.e., *the total output may increase proportionately, more than proportionately and less than proportionately.*
- If increase in the total output is proportional to the increase in inputs – constant returns to scale. For example, if inputs are doubled, then output is also doubled.
- If increase in the output is greater than the proportional increase in the inputs – increasing returns to scale.
- If increase in output is less than proportional to the increase in inputs – diminishing returns to scale.

- The proportion h may be greater than, equal to, or less than k .
- Accordingly, it brings out the three laws of returns to scale.
- If $h=k$, the production function reveals constant returns to scale.
- If $h > k$, the production function reveals increasing returns to scale
- If $h < k$, production function reveals decreasing returns to scale



Production Function and Returns to Scale

- The laws of returns to scale may be explained precisely through the production function.
- Let us assume a production model involving only two variable inputs, K and L and one commodity X. i.e.

$$Q_x = f(K, L)$$

- A production function is said to be homogeneous when all the inputs are increased in the same proportion and this proportion can be factored out.

Laws of Returns to Scale: Input-output relationship through Isoquants

The law of returns to scale may also be presented graphically
Production function of Cobb-Douglas production function

- *The law of returns to scale explain the behaviour of output in response to a proportional simultaneous change in input.*
- *When a firm increases both the inputs proportionately there are three possibilities –*
 - 1. Total output may increase more than proportionately.*
 - 2. Total output may increase proportionately*
 - 3. Total output may increase less than proportionately*

1. Increasing Returns to Scale

1. Increasing Returns to Scale

- Increasing returns to scale is said to operate when the producer is increasing the quantity of all factors in a given proportion, output increases more than proportionately.
- For example, when the quantity of all inputs are increased by 10% and output increases by more than 10%, then we say that increasing returns to scale is operating.
- An increase of labour and capital leads generally to improved organization which increases the efficiency of the work of labour and capital.

- Thus, with every increase in the labour and capital, the total output goes on increasing at an increasing rate and marginal returns and average returns goes on increasing.
- It means if all inputs are doubled output will also increase at the faster rate than double. This increase is due to many reasons like division of labour, specialization and other external economies of scale.

Increasing Returns to Scale

When inputs, K and L are increased at a certain proportion and output increases more than proportionately, it exhibits the law of increasing returns to scale.

For example, if quantities of both the inputs, K and L are successively doubled and the resultant output is more than doubled

Fig 10.17 lines OB and OC represent the expansion path. The movement from point a to b on the line OB means doubling the inputs.

The input combination increases from $1K + 1L$ to $2K + 2L$. As a result of doubling the inputs, output is more than doubled, it increases from 10 to 25 units, i.e., an increase of more than double

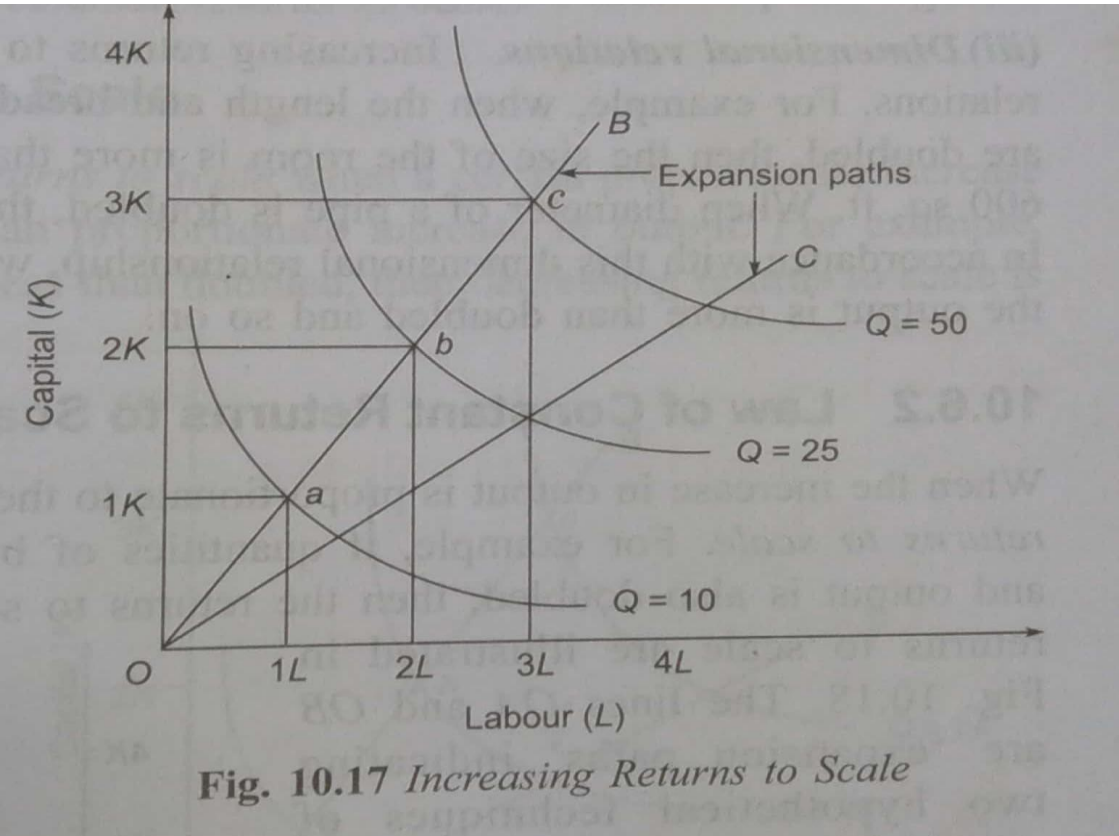


Fig. 10.17 Increasing Returns to Scale

Similarly, the movement from point b to point c indicates 50% increase in inputs as a result of which the output increases from 25 units to 50 units, i.e., by 100%. Clearly, output increases more than the proportionate increase in inputs.

The causes of Increasing Returns to Scale

1. Wider scope for the **use of latest tools**, equipment's, machineries, techniques etc to **increase production and reduce cost per unit**.
2. **Large-scale production** leads to full and complete utilisation of indivisible factor inputs leading to further reduction in production cost
3. As the **size of the plant increases**, more output can be obtained at lower cost
4. As output increases, it is becomes possible to enjoy **several other kinds of economies of scale** like overhead, financial, marketing and risk-bearing economies etc, which is responsible for cost reduction.

2. Constant Returns to Scale

- Constant returns to scale is operating when all factor inputs are increased in a given proportion, output also increases in the same proportion.
- When the quantity of all inputs is increased by 10%, and output also increases exactly by 10%, then we say that constant returns to scale are operating. In this case internal & external economies are exactly equal to internal economies & external diseconomies
- This is also known as *Homogeneous Production Function or Cobb-Douglas Linear homogeneous production function*

Homogeneous Production Function

- A production function is homogeneous when all the inputs are increased in the same proportion and this proportion can be factored out.
- If all the inputs are increased in a certain production and the output increases in the same proportion

$$\begin{aligned} kQ_x &= f(kK, kL) \\ &= k(K, L) \end{aligned} \quad k = \text{Increases}$$

- A homogeneous production function implies constant returns to scale. If K and L are increased by factor k , then Q_x also increases by factor k .

- For example, consider the following production function

$$Q_{x1} = 5K + 10L$$

- If we assume that initially $K=1$, and $L=2$, the total output may be obtained by substituting 1 for K and 2 for L . Thus

$$Q_{x1} = 5(1) + 10(2)$$

$$25 = 5 + 20$$

- If inputs are doubled (i.e., $K=2$ and $L=4$),

$$Q_{x2} = 5(2) + 10(4)$$

$$50 = 10 + 40$$

- **This need not always be true.** It is quite likely that **if all the inputs are increased in a certain proportion, the total output may not increase in the same proportion.**
- **For example, if all the inputs are doubled, the output increases by less than or more than double.**
- **The production function in such cases may be expressed as**

$$hQ_x = f(kK, kL)$$

- **Where h denotes the h -time increase in Q_x as a result of k -time increase in the inputs, K and L .**

- Thus, we find that when inputs are doubled, the output is also doubled.
- Here $K=2$ and since Q_{x2}

$$\frac{Q_{x2}}{Q_{x1}} = \frac{50}{25} = 2$$

$h=2$

- This exhibits constant returns to scale.
- If $h < k$ the production function exhibits the decreasing returns to scale, and if $h > k$, it shows increasing returns to scale

Constant Returns to Scale

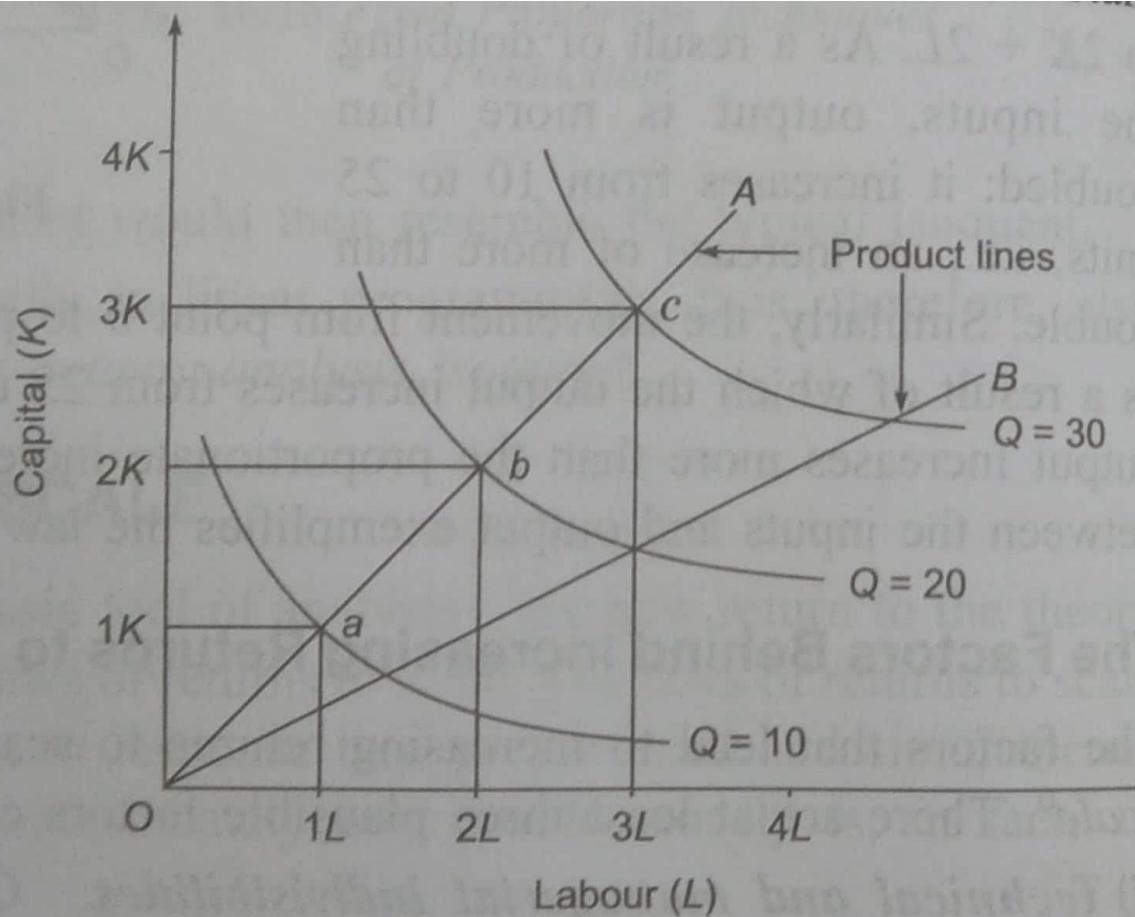


Fig. 10.18 Constant Returns to Scale

Fig 10.18 The lines OA and OB are expansion paths indicating **two hypothetical techniques of production with optimum capital-labour ratio**. The isoquants marked Q=10, Q=20 and Q=30 indicate the three different levels of output.

The movement from points **a** to **b** indicates doubling both the inputs: increasing capital from **1K to 2K** and labour from **1L to 2L**. When inputs are doubled, **output is also doubled**, i.e., output increases from 10 to 20

The movement from point **b** to **c** indicates a **50%** increase in both labour and capital. This increase in inputs results in an increase of output from **20 to 30** units, i.e., a **50%** increase in output

$$1K + 1L \Rightarrow 10$$

$$2K + 2L \Rightarrow 20$$

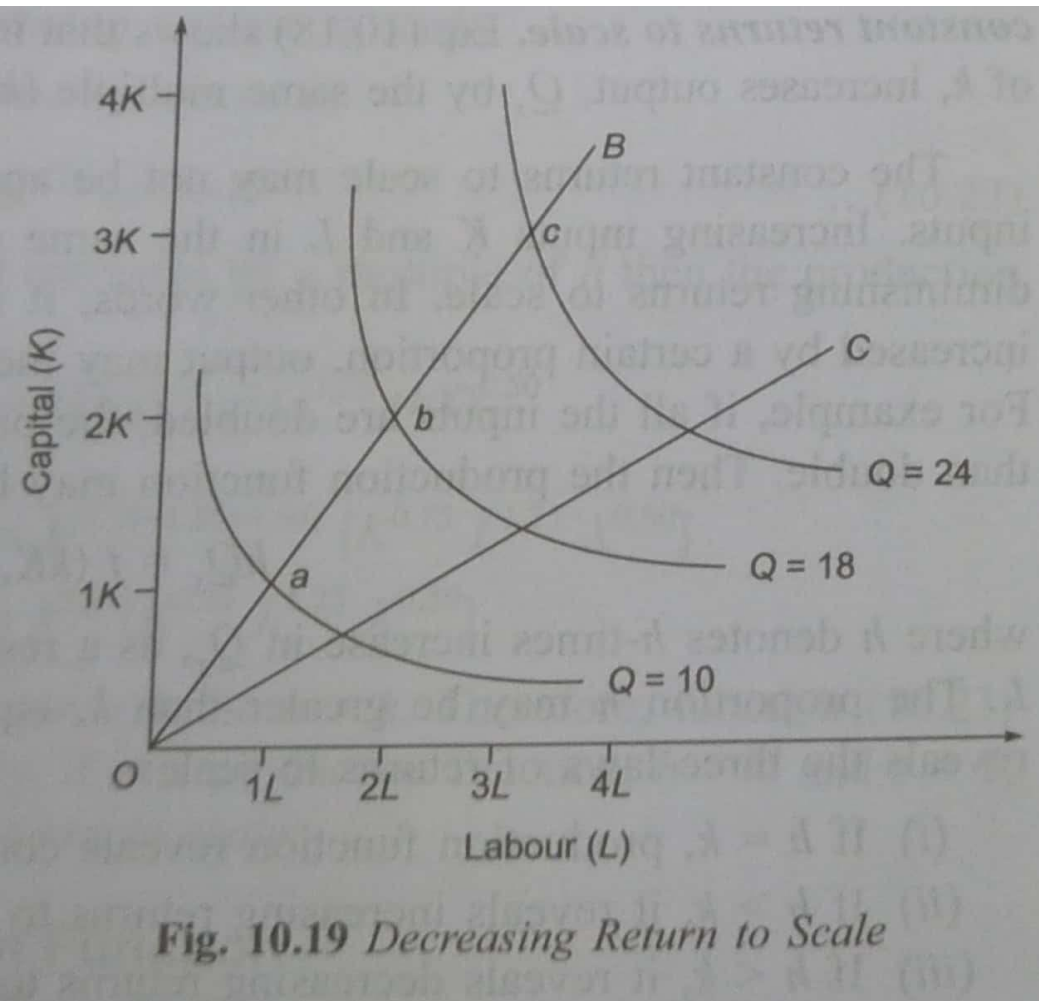
$$3K + 3L \Rightarrow 30$$

3. Decreasing Returns to Scale

3. Diminishing Returns to Scale

- DRS or increasing costs refers to that production situation, where if all the factors of production are increased in a given proportion, output increases in a smaller proportion.
- Marshall defines “*An increase in the capital and labour applied in the cultivation of land causes in general a less than proportionate increase in the amount of the produce raised, unless it happens to coincide with an improvement in the art of agriculture*”.
- It means, if inputs are doubled, output will be less than proportionate.

Decreasing Returns to Scale



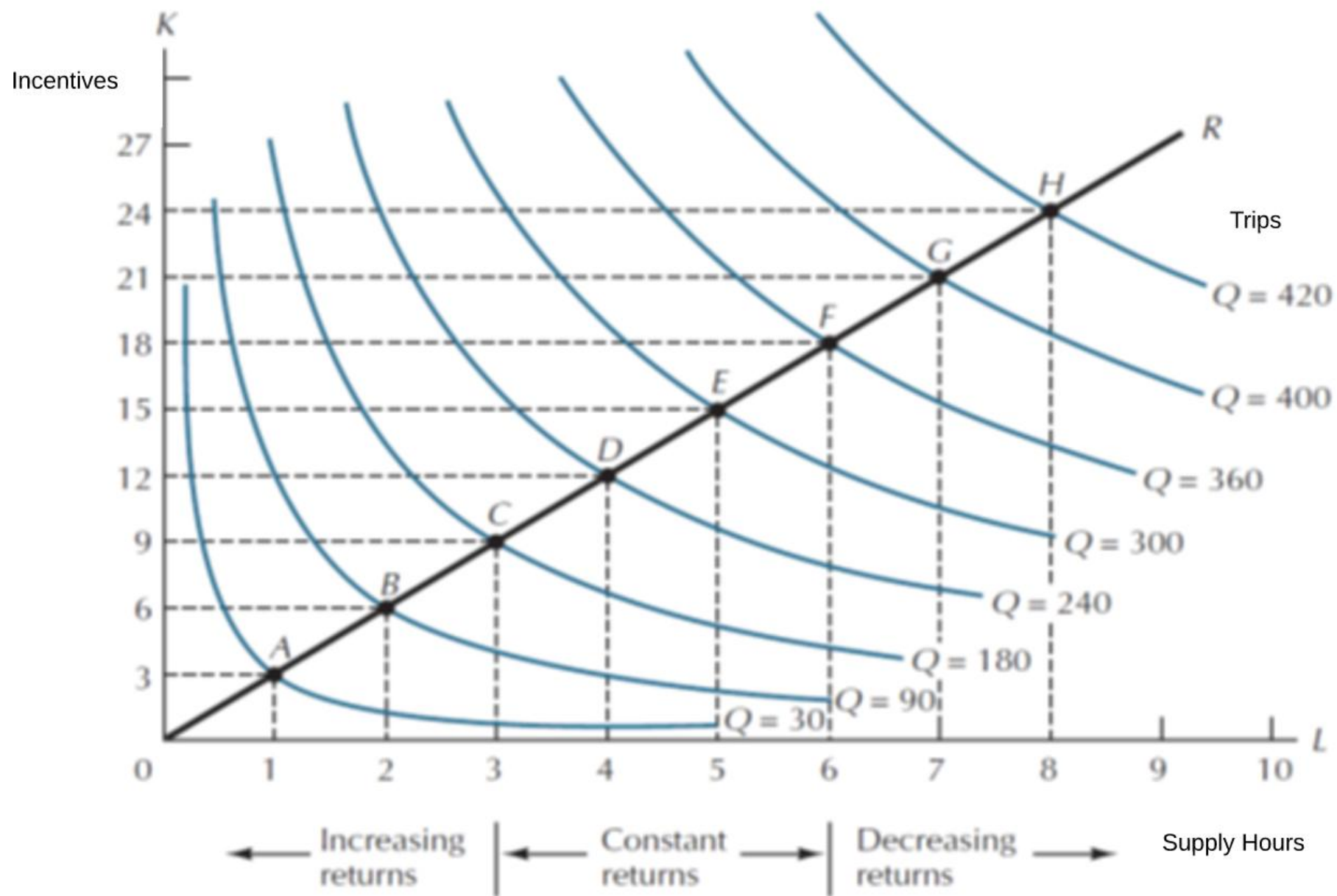
When a certain proportionate increase in inputs, K and L , leads to a less than proportionate increase in output. For example, when inputs are doubled and output is less than doubled, then decreasing returns to scale is in operation.

When the inputs K and L are doubled, i.e., when capital-labour combination is increased from $1K + 1L$ to $2K + 2L$, the output increases from 10 to 18 units. This means that when capital and labour are increased by 100% output increases by only 80%. That is, increasing output is less than the proportionate increase in inputs.

Similarly, movement from point b to c indicates a 50% increase in the inputs. But the output increases by only 33.3%. This exhibits decreasing returns to scale.

The causes of diminishing returns to scale

1. Emergence of difficulties in co-ordination and control
2. Difficulty in effective and better supervision
3. Delays in management decisions
4. Inefficient and miss-management due to over growth and expansion of the firm
5. Productivity and efficiency declines unavoidably after a point.



$$Q = K^{0.25} L^{0.50} \quad \dots(10.21)$$

If K and L are multiplied by k , and output increases by a multiple of h then

$$hQ = (kK)^{0.25} (kL)^{0.50}$$

By factoring out k , we get

$$\begin{aligned} hQ &= k^{0.25+0.50} [K^{0.25} L^{0.50}] \\ &= k^{0.75} [K^{0.25} L^{0.50}] \end{aligned} \quad \dots(10.22)$$

In Eq. (10.22), $h = k^{0.75}$ and $r = 0.75$. This means that $r < 1$ and, thus, $h < k$. Production function (10.21), therefore, shows ***decreasing returns to scale***.

Now consider another production function given as

$$Q = K^{0.75} L^{1.25} X^{0.50} \quad \dots(10.23)$$

If K , L and X are multiplied by k , Q increases by a multiple of h then the production function can be expressed as

$$hQ = (kK)^{0.75} (kL)^{1.25} (kX)^{0.50}$$

By factoring out k , we get

$$\begin{aligned} hQ &= k^{(0.75+1.25+0.50)} [K^{0.75} L^{1.25} X^{0.50}] \\ &= k^{2.5} [K^{0.75} L^{1.25} X^{0.50}] \end{aligned}$$

Here $h = k^{2.5}$ where $2.5 = r$. It means $r > 1$. So $h > k$. Therefore, function (10.23) gives ***increasing returns to scale***. Similarly, if in a production function, $h = k^r$ and $r = 1$, the production function gives ***constant returns to scale***.